

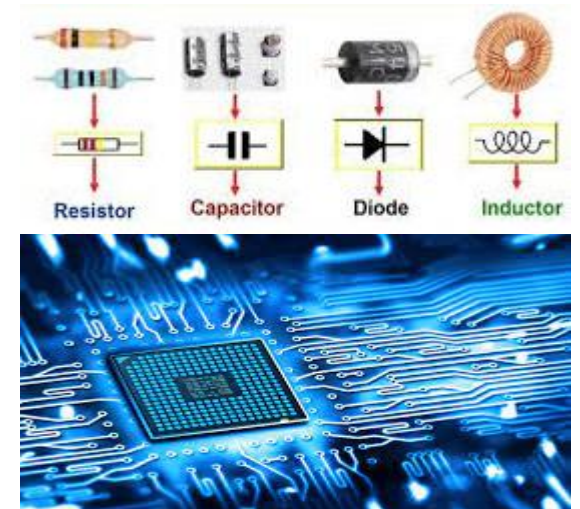


Electronics 1

BSC 113

Summer 2021-2022

Lecture 5



Mesh analysis & Superposition Methods

INSTRUCTOR

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➤ Contents

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- 4) CASE 2
- 5) Superposition

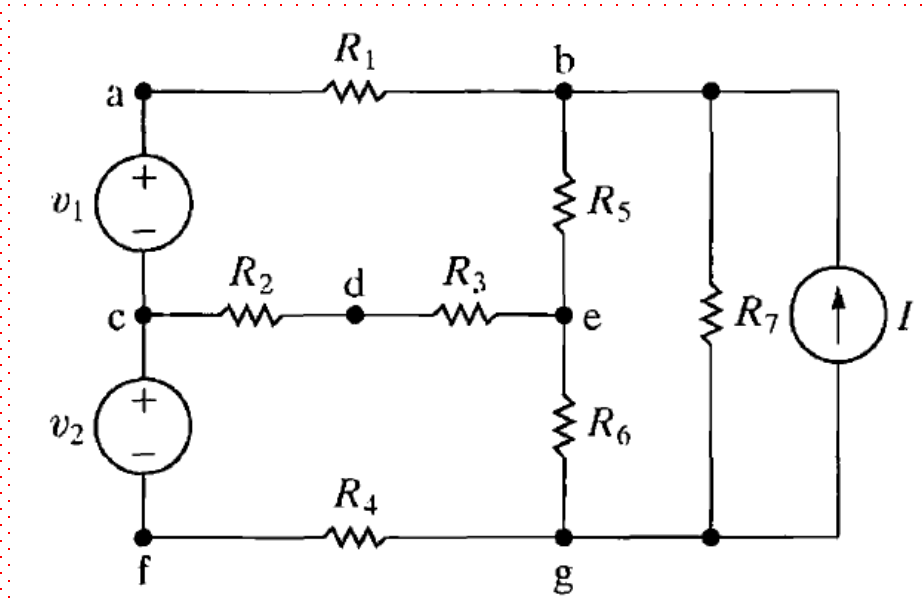


➤ Terms of describing circuits

Name	Definitions
Node	A point where two or more circuit elements join
Essential node	A node where three or more circuit elements join
Branch	A path that connects two nodes
Essential branch	A path which connects two essential nodes without passing through an essential node
Loop	A path whose last node is the same as the starting node
mesh	A loop that does not enclose any other loops

Example 1

- For the circuit in the figure, identify
 - a) all nodes.
 - b) all essential nodes.
 - c) all branches.
 - d) all essential branches.
 - e) all meshes.
 - f) two loops that are not meshes.



Example 1

a) The nodes are a, b, c, d, e, f, and g.

b) The essential nodes are b, c, e, and g.

c) The branches are v_1 , v_2 , R_1 , R_2 , R_3 , R_4 , R_5 , R_6 , R_7 and I .

d) The essential branches are:

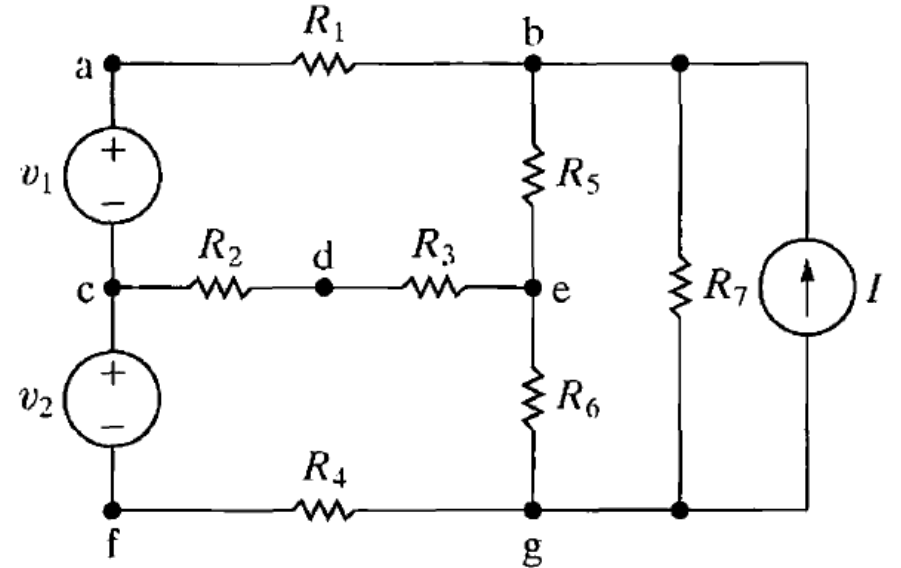
$$v_1 - R_1,$$

$$R_2 - R_3,$$

$$v_2 - R_4,$$

$$R_5, R_6, R_7 \text{ and}$$

$$I$$



Example 1

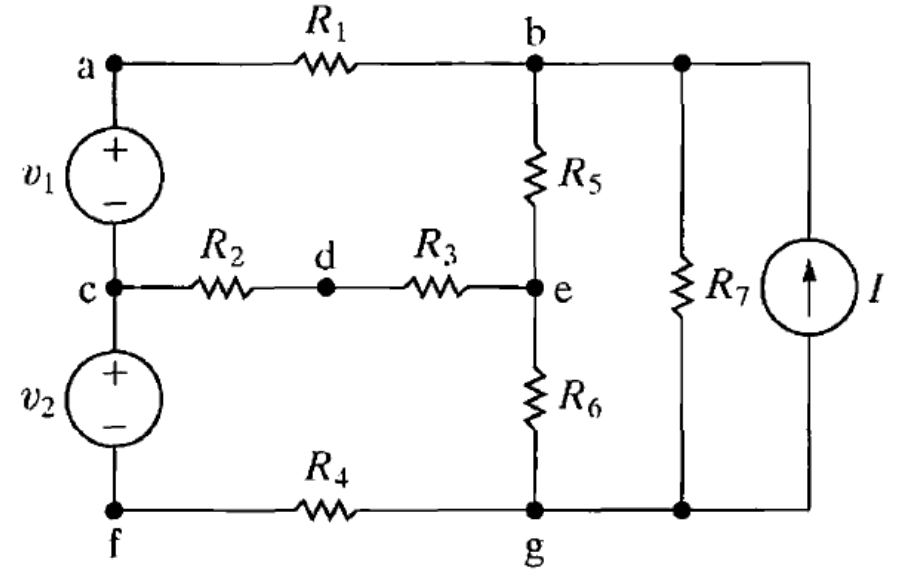
e) The meshes are:

$$v_1 - R_1 - R_5 - R_3 - R_2,$$

$$v_2 - R_2 - R_3 - R_6 - R_4,$$

$$R_5 - R_7 - R_6 \text{ and}$$

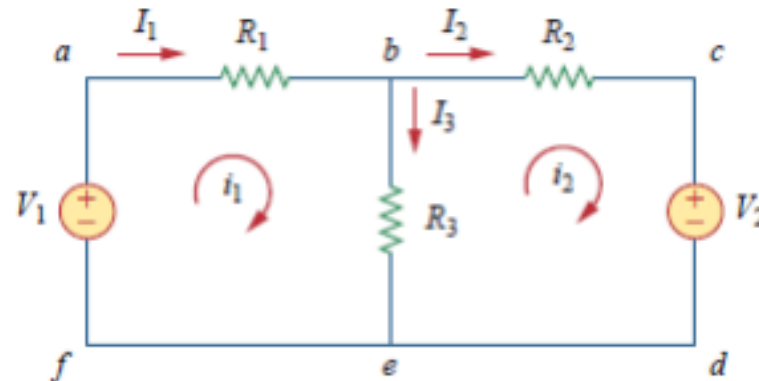
$$R_7 - I$$



f) The two loops that are not meshes are $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ and $I - R_5 - R_6$, because there are two loops within them.

□ Mesh analysis

- In the mesh analysis of a circuit with n meshes, we take the following three steps.
1. Assign mesh currents i_1, i_2, \dots, i_n to the n meshes.
 2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
 3. Solve the resulting n simultaneous equations to get the mesh currents.



□ Mesh analysis

➤ As shown in figure 2.9,

loop 1:

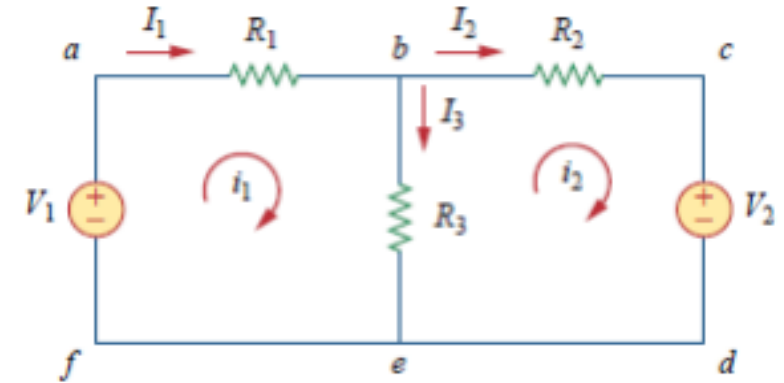
$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

loop 2:

$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

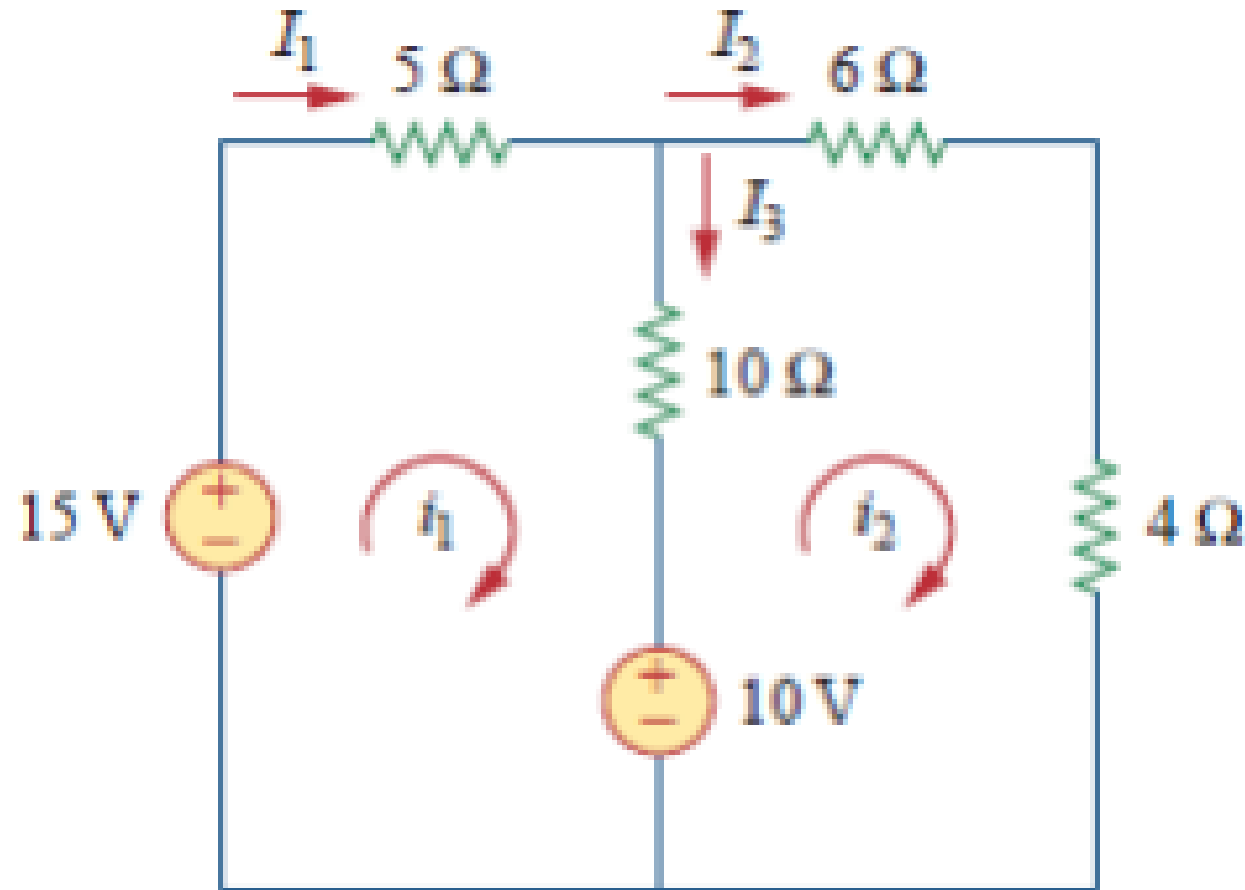
After we will solve the two equation we can find:

$$I_1 = i_1, \quad I_2 = i_2 \quad \text{and} \quad I_3 = i_1 - i_2$$



□ Example 2

- Find the branch currents I_1 , I_2 and I_3 using mesh analysis.



□ Example 2 solution:

➤ Answer: We first obtain the mesh currents using KVL.

For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \quad (1)$$

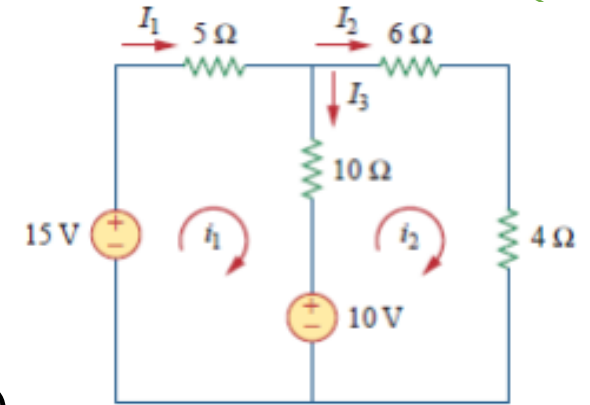
and for mesh 2

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \quad (2)$$

from (1) and (2)

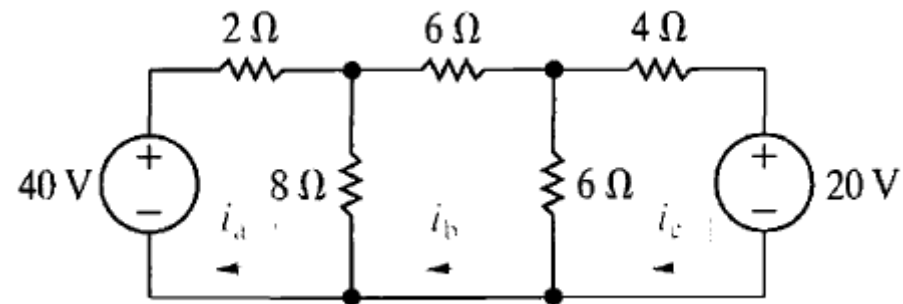
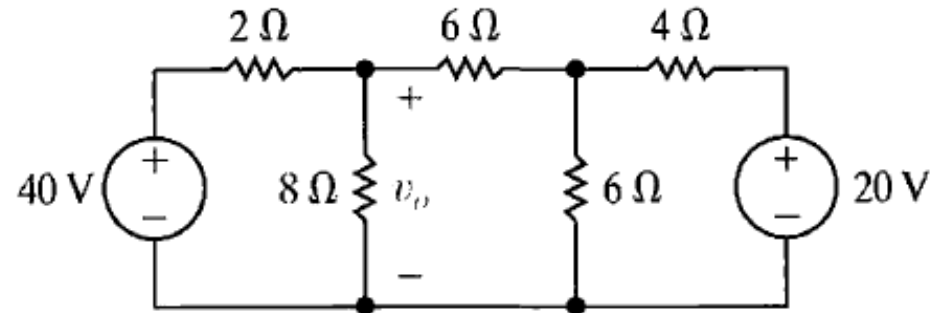
$$i_1 = i_2 = 1A$$

$$I_1 = i_1 = 1A, \quad I_2 = i_2 = 1A \quad \text{and} \quad I_3 = i_1 - i_2 = 0A$$



Example 3

- Use the mesh-current method to determine the power associated with each voltage source in the circuit shown.
- Calculate the voltage v_o across the $8\ \Omega$ resistor.



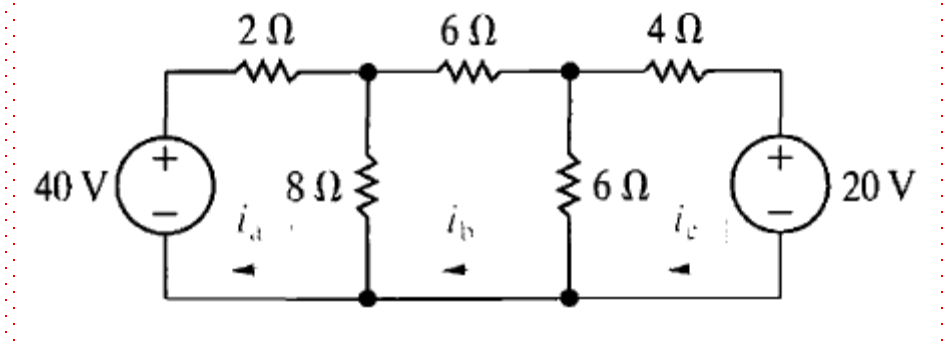
Example 3

a) The three meshes equations are:

$$-40 + 2i_a + 8(i_a - i_b) = 0$$

$$8(i_b - i_a) + 6i_b + 6(i_b - i_c) = 0$$

$$6(i_c - i_b) + 4i_c + 20 = 0$$



Therefore, the three mesh currents are

$$i_a = 5.6 \text{ A,}$$

$$i_b = 2.0 \text{ A,}$$

$$i_c = -0.80 \text{ A.}$$

The power associated with each voltage source:

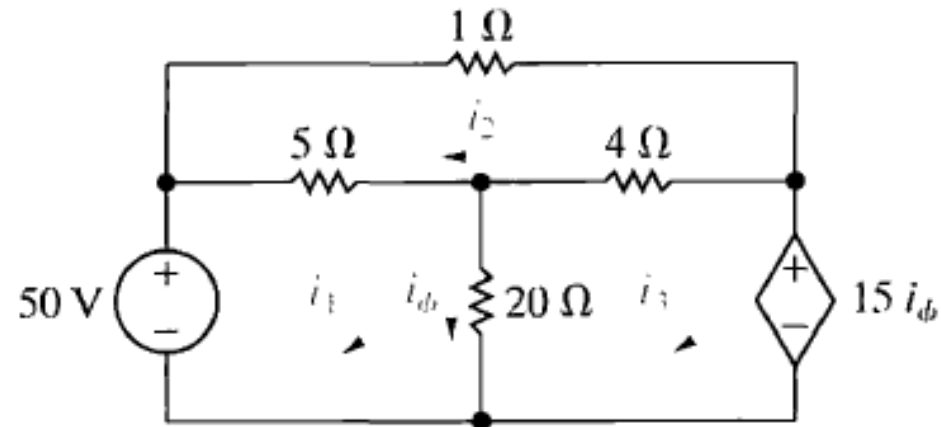
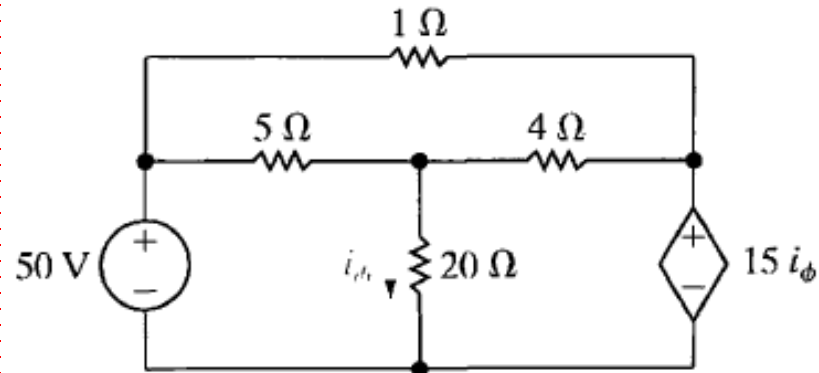
$$P_{40\text{V}} = -40i_a = -224 \text{ W,}$$

$$P_{20\text{V}} = 20i_c = -16 \text{ W.}$$

b) $v_o = 8(i_a - i_b) = 8(3.6) = 28.8 \text{ V.}$

Example 4

Use the mesh-current method of circuit analysis to determine the power dissipated in the 4Ω resistor in the circuit shown.



Example 4

- The three mesh-current equations are:

$$5(i_1 - i_2) + 20(i_1 - i_3) = 50$$

$$5(i_2 - i_1) + i_2 + 4(i_2 - i_3) = 0$$

$$20(i_3 - i_1) + 4(i_3 - i_2) + 15i_\Phi = 0$$

- We now express the branch current controlling the dependent voltage source in terms of the mesh currents as:

$$i_\Phi = i_1 - i_3$$

- Therefore, the mesh currents are:

$$i_1 = 29.6$$

$$i_2 = 26 \text{ A}$$

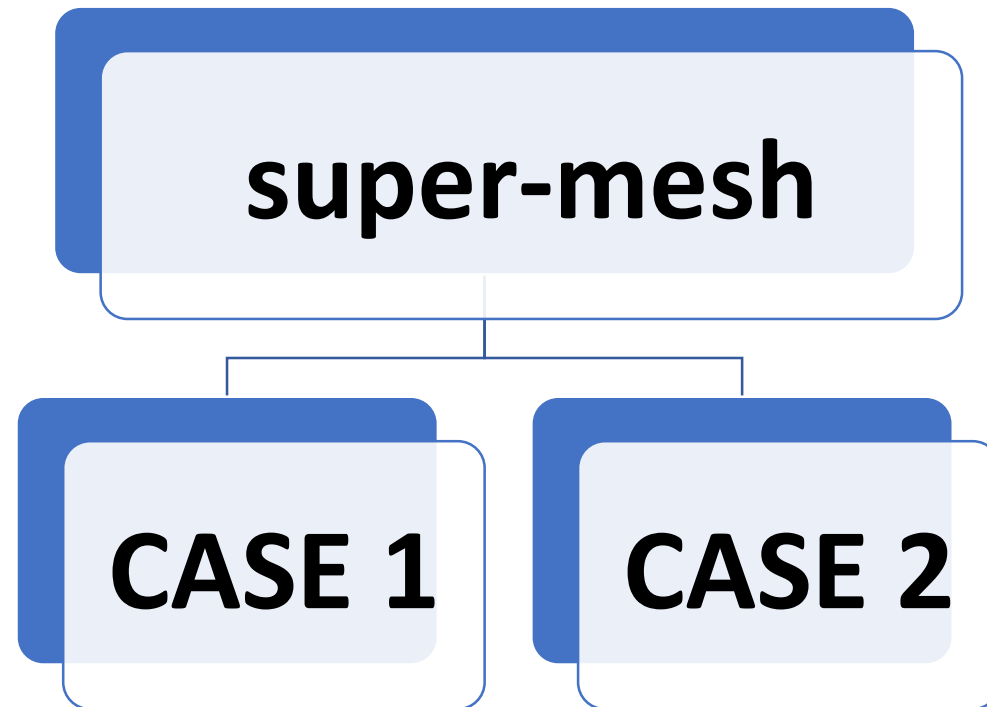
$$i_3 = 28 \text{ A}$$

and

$$P_{4\Omega} = (i_3 - i_2)^2(4) = (2)^2(4) = 16 \text{ W.}$$

❑ super-mesh

- Mesh Analysis with Current Sources is called super-mesh (A super-mesh results when two meshes have a (dependent or independent) current source in common.) and considers as special case.

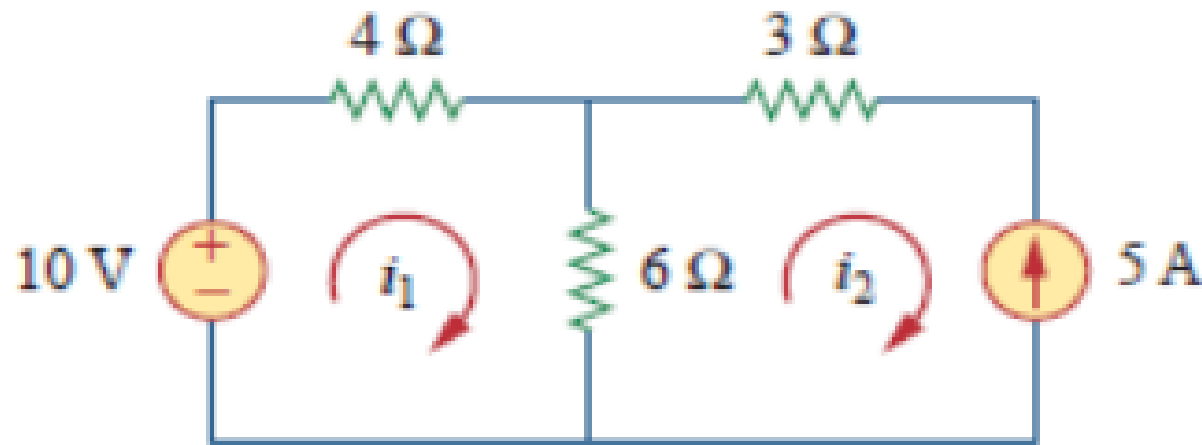


□ CASE 1

- When a current source exists only in one mesh:
- Consider the circuit in next figure, for example. We set $i_2 = -5 \text{ A}$ and write a mesh equation for the other mesh in the usual way; that is,

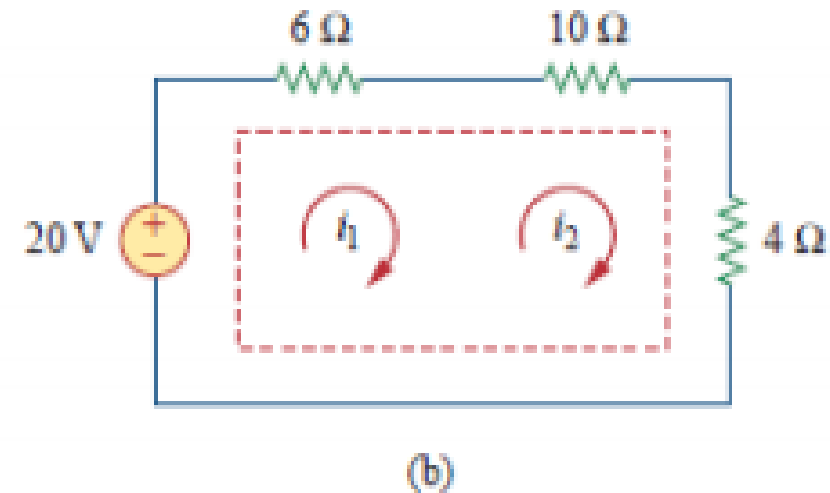
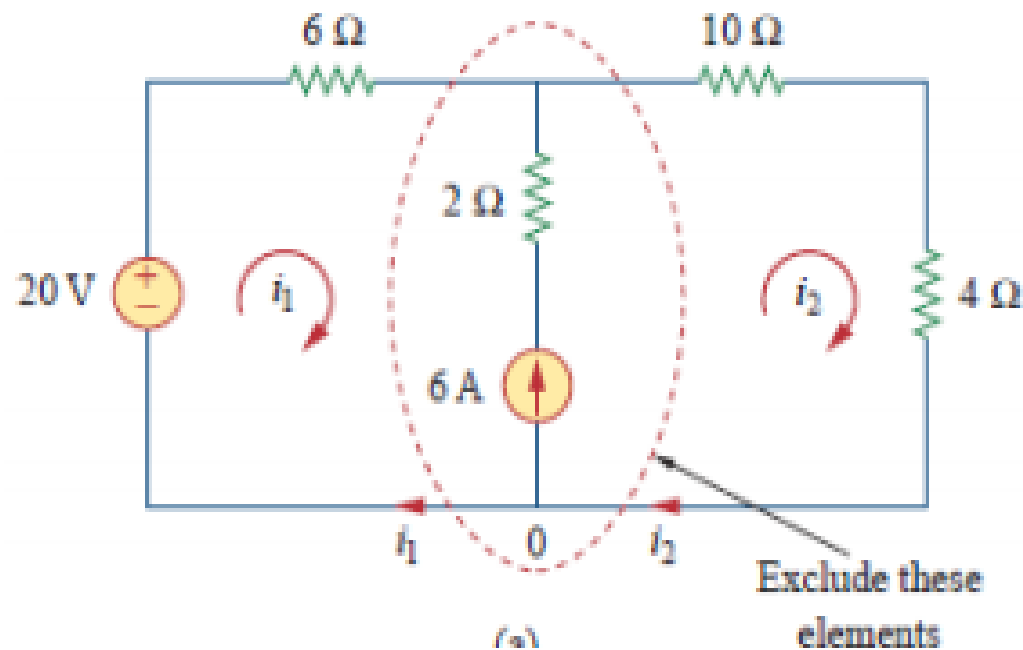
$$-10 + 4i_1 + 6(i_1 - (-5)) = 0 \rightarrow i_1 = -2\text{A}$$

- Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.

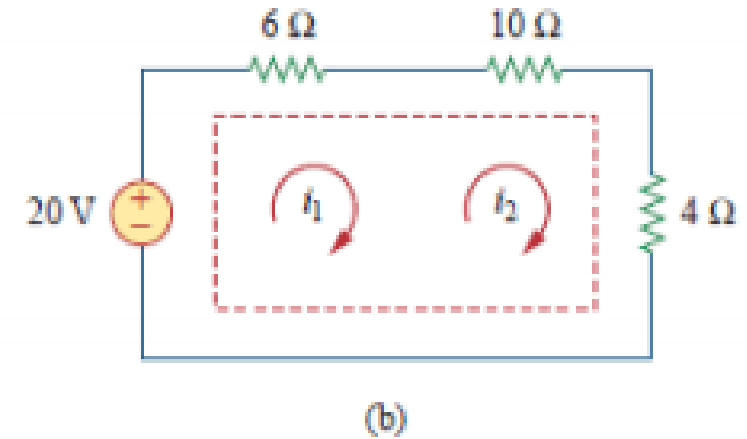
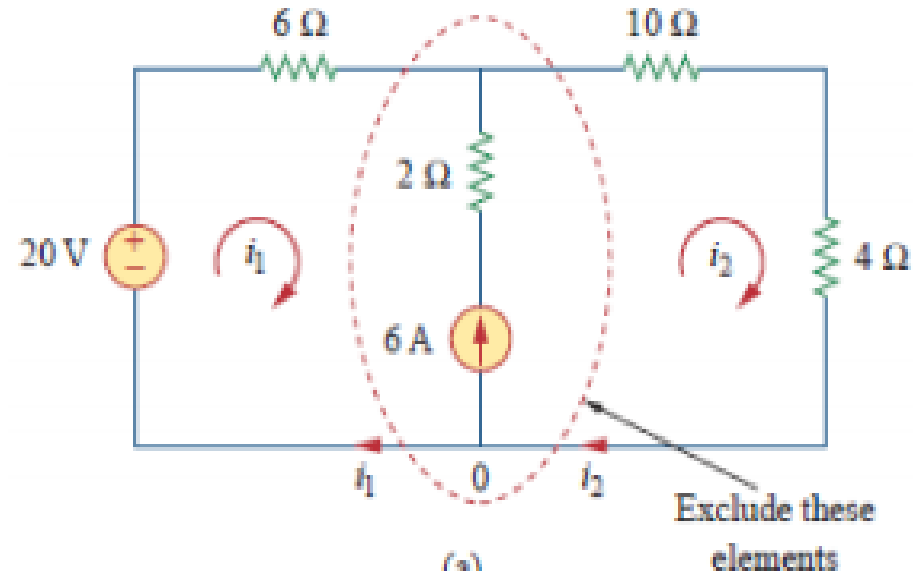


□ CASE 2

- When a current source exists between two meshes:
- Consider the circuit in next figure, for example. We create a super-mesh by excluding the current source and any elements connected in series with it.



□ CASE 2



$$i_2 - i_1 = 6 \quad (1)$$

$$-20 + 6 i_1 + 10 i_2 + 4 i_2 = 0 \quad (2)$$

from (1) and (2)

$$i_1 = -3.2 \text{ A}, \quad i_2 = 2.8 \text{ A}$$

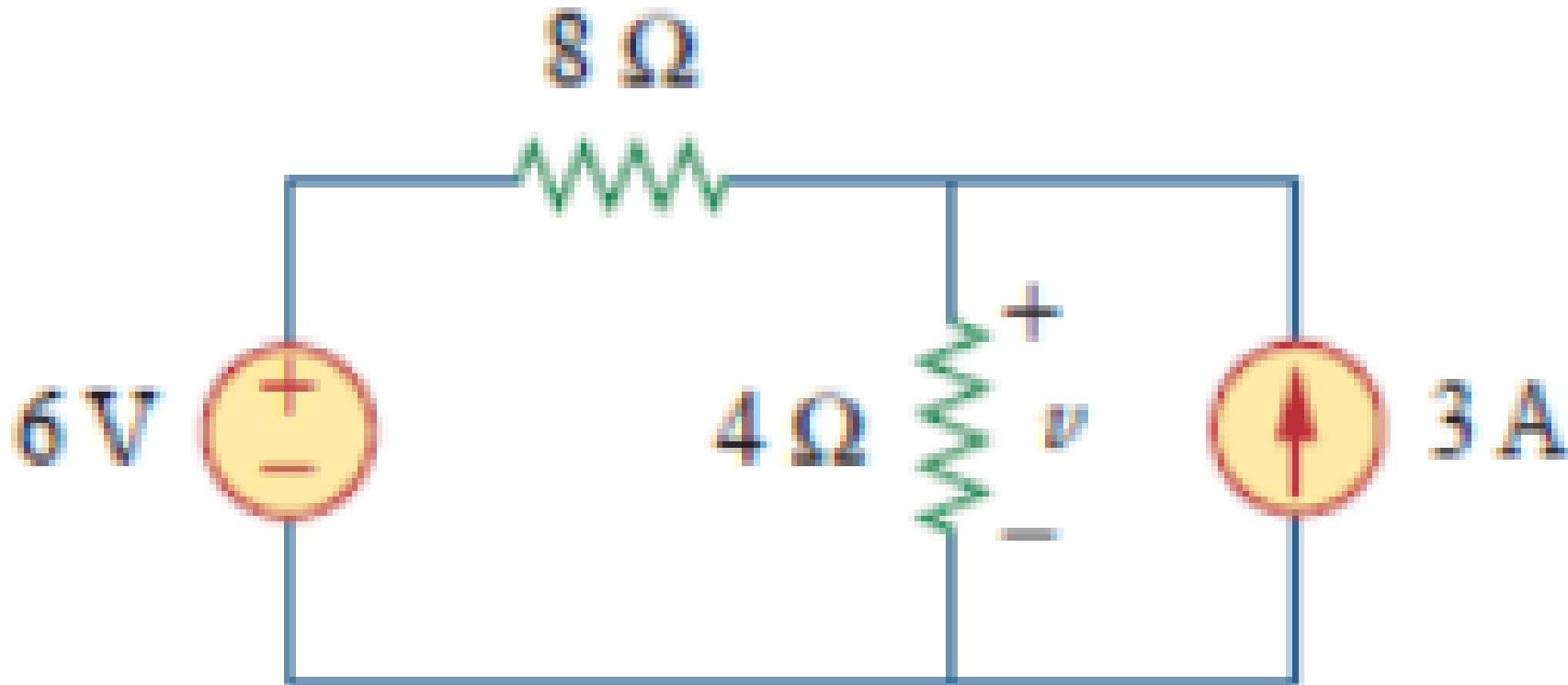
Superposition Method

□ Superposition

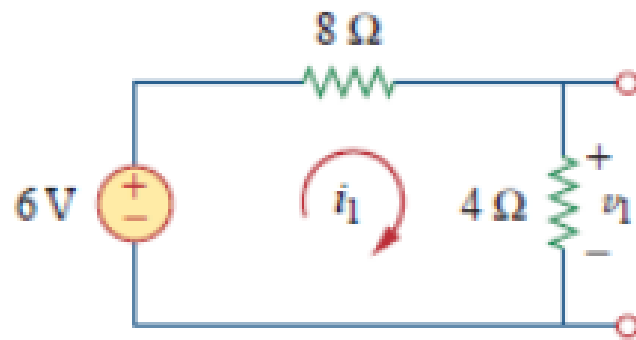
- The superposition principle states that the voltage across (or current through) an element in a linear circuit **is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone**. With these in mind, **we apply the superposition principle in three steps**:
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
 2. Repeat step 1 for each of the other independent sources.
 3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

□ Example 5:

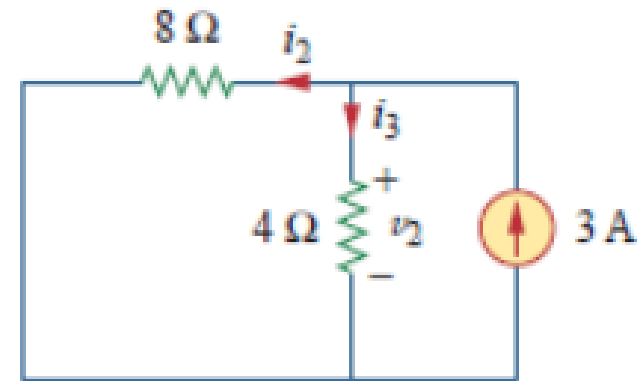
- Use the superposition theorem to find v in the circuit



□ Example5:



(a)



(b)

Answer: $v = v_1 + v_2$

from figure a by voltage divider

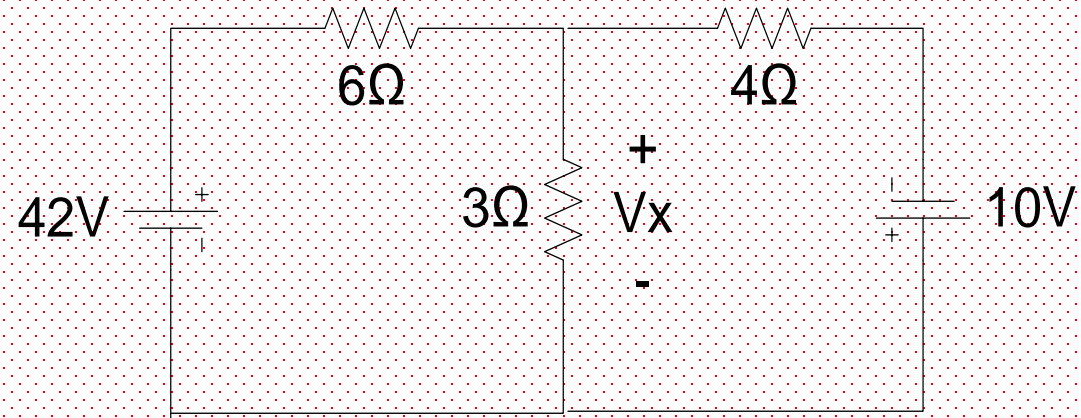
$$v_1 = 4i_1 = \frac{4}{4 + 8} * 6 = 2V$$

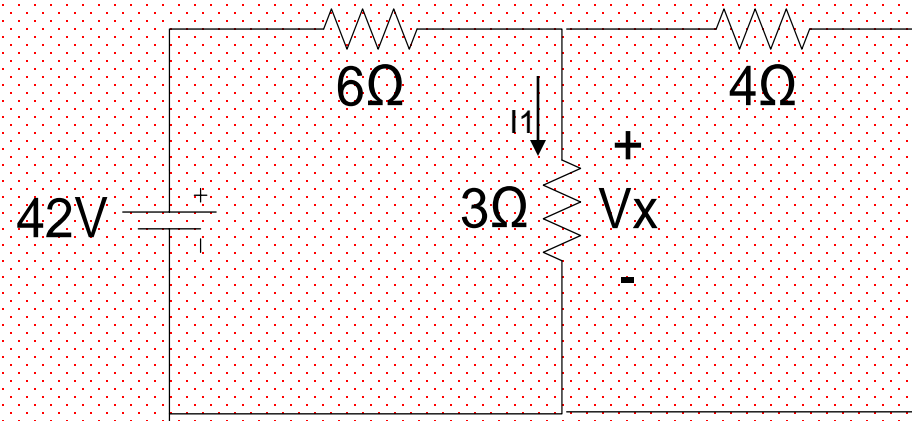
from figure b by current divider

$$v_2 = 4i_3 = 4 * \frac{8}{4 + 8} * 3 = 8V$$

$$v = v_1 + v_2 = 2 + 8 = 10V$$

Example 6: Find voltage V_x using superposition theorem

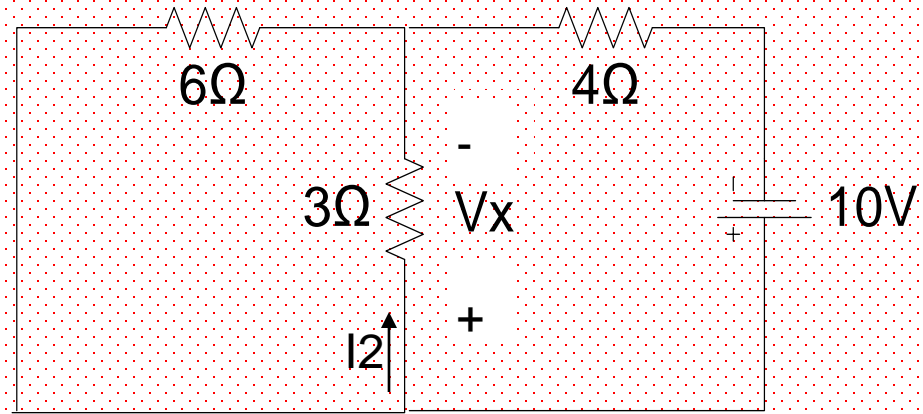




Considering 42V source only (10V source SC)

$$V_{x(42V)} = \frac{(3 \parallel 4)}{6 + (3 \parallel 4)} \times 42 = \frac{(12/7)}{6 + (12/7)} \times 42$$

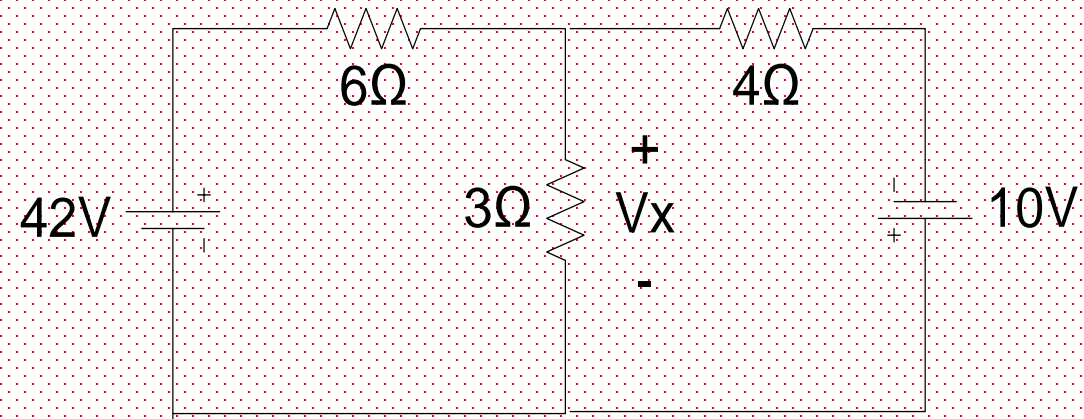
$$= 9.333V$$



Only 10V source connected (42V source replaced by SC)

$$V_{x(10V)} = -\frac{(6 \parallel 3)}{(6 \parallel 3) + 4} \times 10 = -\frac{2}{2 + 4} \times 10$$

$$= -3.333V$$

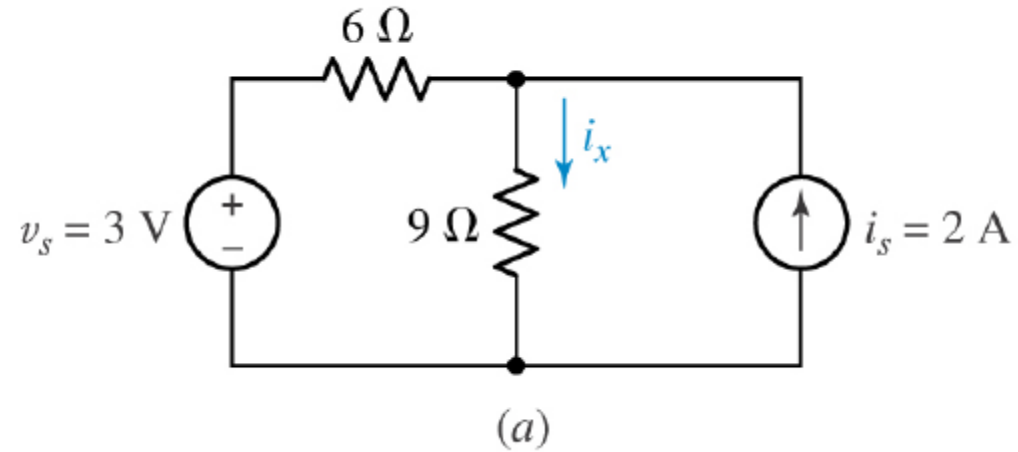


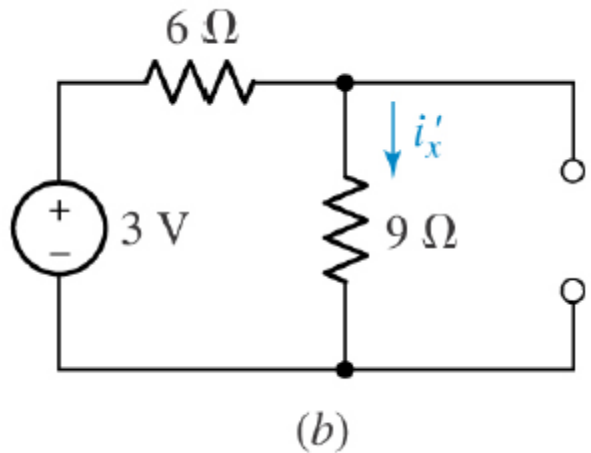
Total Voltage =

$$V_x = V_{x(42V)} + V_{x(10V)}$$

$$= 9.333 - 3.333 = 6V$$

Example 7: Use superposition to find i_x

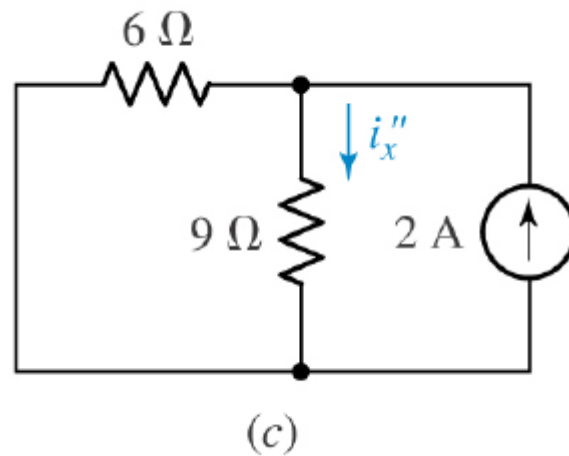




Step 1:

Only 3V source connected (2A source is OC)

$$i'_x = 3/15 = 0.2 \text{ A}$$



Step 2:

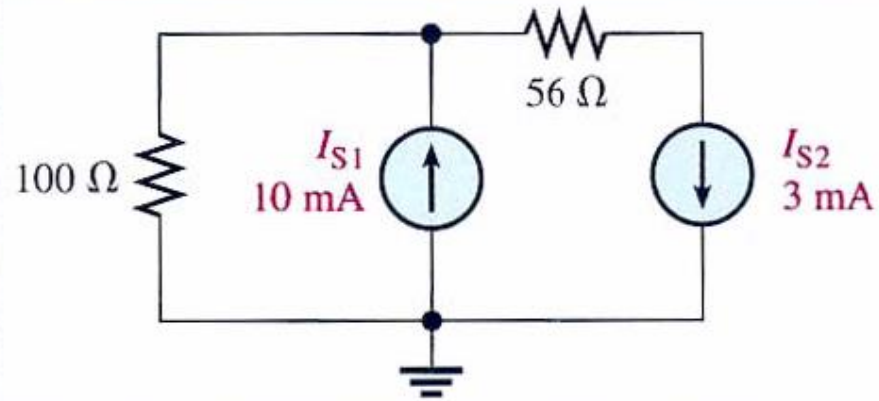
Only 2A source connected (3V source is SC)

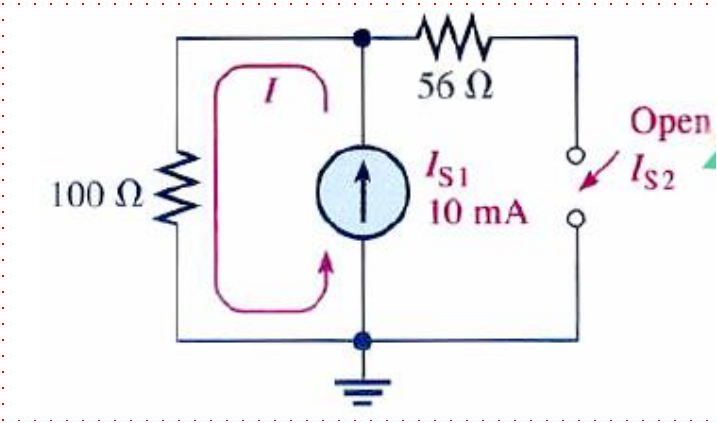
$$i''_x = 2 \times 6 / (6 + 9) = 0.8 \text{ A}$$

Step 3: Total current = 0.2 + 0.8 = 1A

$$i_x = 1.0 \text{ A}$$

EXAMPLE Find the current through the $100\ \Omega$ resistor.

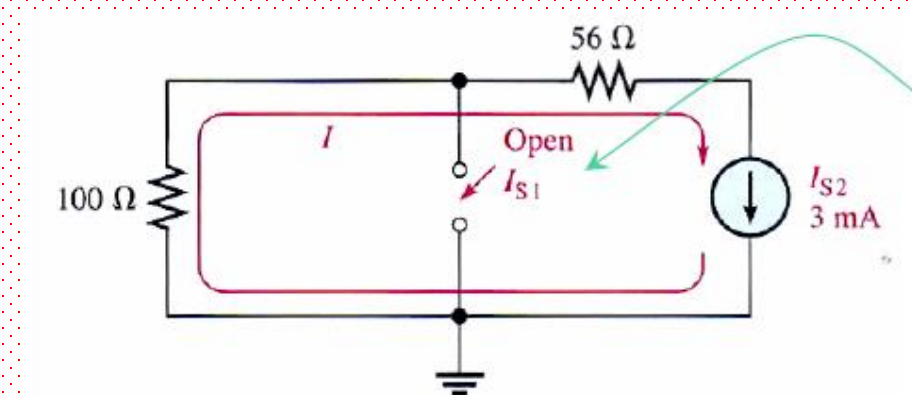




Step 1:

Only 10mA source connected in circuit(3mA source OC)

$I=10\text{mA}$



Step2: Only 3mA source connected(10mA source OC)

$I'=3\text{mA}$

Total Current: $10\text{mA}-3\text{mA}=7\text{mA}$

Thank
you

